Exact Design of TEM Microwave Networks Using Quarter-Wave Lines*

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Summary—Modern network theory procedures, based on Ozaki-Ishii[1] synthesis techniques, are reviewed for application in the design of TEM mode microwave networks using parallel coupled bars and/or series and shunt stubs. The circuit equivalences and identities obtained are theoretically valid over the entire frequency spectrum and lead to several physical configurations having identical response functions. These equivalent circuits often allow simplification of the physical circuitry and realization of both broad and narrow bandwidths. The problem of practical circuit configurations is discussed from the viewpoint of bandwidth and circuit element values. Neglecting multiple responses, TEM low-pass, high-pass and band-pass Butterworth filters are shown to offer steeper bandedge characteristics than those of corresponding lumped element filters. The use of complementary filters to match a source and load over a wide frequency range is outlined and TEM realizations of these complements are obtained. A simple procedure for obtaining element values of Butterworth complements is described. An analysis of parallel coupled filters is made and a simplified equivalent circuit is obtained. An exact synthesis procedure for parallel coupled bar filters and their equivalent forms is given. Construction details and experimental results are described for two filters which use series stubs.

I. Introduction

THE PURPOSE of this paper is to present a unified account of an exact modern network synthesis applicable to the design of distributed TEM mode networks. Emphasis is placed on microwave circuit aspects rather than network theory wherever possible. In the past, most TEM microwave networks have been designed by use of image parameter methods. The depth of understanding of lumped element circuits afforded by modern network theory has not been completely extended to distributed networks. A discussion of the relative merits of modern network and image parameter network theories can be found in literature.[2,3] The details of modern network synthesis are beyond the scope of this paper and may be reviewed in existing publications.[2,4,5]

This paper is presented as follows: Section II introduces the transformation that allows distributed TEM networks to be treated in a manner analogous to lumped element networks. Equivalent circuits for various configurations are derived and tabulated. The "unit element" is introduced as a necessary circuit element and Kuroda's identities are discussed. The section concludes with a design example and a discussion of impedance and frequency scaling.

Section III describes a number of equivalent physical structures that realize the same circuit response. Achievement of practical bandwidths and element values is related to the circuit equivalences.

Section IV contains a comparison between lumped element and distributed filter characteristics. The repetitive nature of the microwave circuits is not considered. Under this condition an analysis of the ability of each type of network to approximate the ideal filter characteristic is given.

Section V applies the theory of the preceding sections to some interesting network problems. Parallel coupled line filters are analyzed and equivalent circuits are obtained. Complementary microwave filters are introduced and network realizations are obtained. Construction details and experimental results are given for two filters containing series stubs.

Appendix I demonstrates the methods for exact filter synthesis of uniform parallel coupled bars, lines with shunt shorted stubs and lines with series open stubs. Appendix II outlines a procedure for eliminating ideal transformers in networks having parallel coupled bars.

II. Modern Network Filter Theory

The basis for modern network theory was established in the mid-1920's with the work of Foster and Cauer and was culminated with the conditions for realizability of driving-point impedances stated by Brune in 1930.[6] From a consideration of the energy functions associated with a LLFPB (lumped, linear, finite, passive and bilateral) network, Brune showed that, for a function of a complex variable to be a realizable driving point impedance, it is necessary and sufficient that it be a "positive real" function—where a positive real (p.r.) function is one that

1) is real for real values of the complex frequency variable $s=\sigma+j\omega$ and
2) has a positive real part for values of s having a positive real part.

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[1] H. Ozaki and J. Ishii, "Synthesis of a class of strip-line filters," IRE Trans. on Circuit Theory, vol. CT-5, pp. 104–109; June, June, 1958.

[2] L. Weinberg, "Network Analysis and Synthesis," McGraw-Hill Book Co., Inc., New York, N. Y.; 1962

[3] "Reference Data for Radio Engineers," published by International Telephone and Telegraph Corp., 4th ed., pp. 187–188; 1957.

[4] E. A. Guillemin, "Synthesis of Passive Networks," John Wiley and Sons, Inc., New York, N. Y.; 1957.

[5] N. Balabanian, "Network Synthesis," Prentice-Hall, Inc., Englewood Cliffs, N.J.; 1958.

[6] O. Brune, "Synthesis of a finite 2-terminal network whose driving point impedance is a prescribed function of frequency," *J. Math. and Phys.*, vol. 10, pp. 191–236; 1930.

From these two conditions, it is possible to determine if a given impedance function of s can be realized.

The extension of lumped network synthesis techniques to microwave networks was demonstrated by Richards[7] in 1948. In microwave networks, the impedances of sections of transmission line are expressed as functions of $j \tan \beta l$ where β is the phase constant along the transmission line and l is the line length. Richards showed that microwave networks composed of lumped resistors and lossless equal length transmission lines can be treated as lumped networks using a transformation to the complex frequency variable

$$S = j \tan \frac{\pi f}{2f_0} \tag{1}$$

where f_0 is the real constant frequency at which l is a quarter wavelength and f is the real frequency variable. As can be seen, S is periodic in $2f_0$ and the response of a microwave network will repeat at this interval. The mapping properties of functions of the variable S are illustrated in Fig. 1 for the case of a high pass S-plane filter.

The synthesis procedure to be used becomes apparent from Fig. 1.[8] The required response is synthesized using lumped elements in the S plane and is then converted to a microwave structure in the f plane. The only remaining problem is to express the microwave structures as functions of S. One TEM mode structure to be considered for application in modern network design is shown in Fig. 2.

Jones and Bolljahn[9] have derived the impedance matrix for the configuration of Fig. 2(a).

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = -j(Z_{0e} + Z_{0o}) \frac{\cot \theta}{2}$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = -j(Z_{0e} - Z_{0o}) \frac{\cot \theta}{2}$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = -j(Z_{0e} - Z_{0o}) \frac{\csc \theta}{2}$$

$$Z_{14} = Z_{41} = Z_{23} = Z_{32} = -j(Z_{0e} + Z_{0o}) \frac{\csc \theta}{2}$$

[7] P. I. Richards, "Resistor transmission-line circuits," Proc. IRE, vol. 36, pp. 217–220; February, 1948.

[8] The basic synthesis procedure used follows that of Ozaki and Ishii.[1]

[9] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission line filters and directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. 19, pp. 75–81; April, 1956.

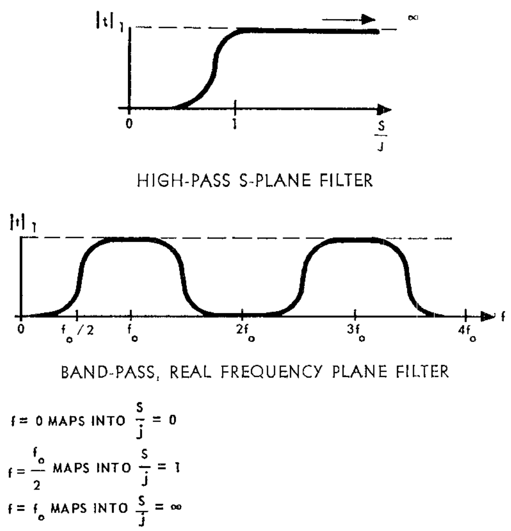


Fig. 1—Mapping properties of a function of the variable $S/j = \tan \pi f/2f_0$.

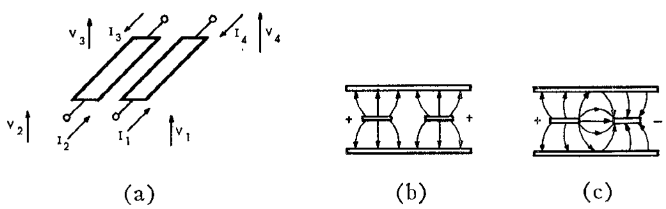


Fig. 2—Parallel coupled stripline four-port. (a) Four-port parameters. (b) Even mode field configuration. (c) Odd mode field configuration.

where

θ = electrical length of the line,
Z_{0e} = even mode impedance,
Z_{0o} = odd mode impedance.

The above relations hold for equal width strips. The matrix for unequal strip widths, as well as methods for calculating even and odd mode impedances, are given by Ozaki and Ishii.[1] By using unequal strip widths, an additional degree of freedom is obtained as required in many practical cases. The analysis of unequal width strips follows the same procedure as that used for equal width strips. For simplicity, the derivation will be carried out for the equal width case. Substituting

$$S = j \tan \frac{\pi f}{2f_0} = j \tan \theta \text{ gives}$$

$$Z_{11} = Z_{22} = Z_{33} = Z_{44} = \frac{Z_{0e} + Z_{0o}}{2S}$$

$$Z_{12} = Z_{21} = Z_{34} = Z_{43} = \frac{Z_{0e} - Z_{0o}}{2S}$$

$$Z_{13} = Z_{31} = Z_{24} = Z_{42} = \frac{Z_{0e} - Z_{0o}}{2S} \sqrt{1 - S^2}$$

$$Z_{14} = Z_{41} = Z_{23} = Z_{32} = \frac{Z_{0e} + Z_{0o}}{2S} \sqrt{1 - S^2}. \tag{2}$$

To determine the impedance parameters when the two-strip configuration is used as a twoport, it is only necessary to apply the pertinent port conditions. For example, assume that terminal (2) is grounded, (3) is open, (1) is the input port and (4) is the output port. The port conditions are then $V_2=0$ and $I_3=0$. It follows that

$$\begin{bmatrix} V_1 \\ 0 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ 0 \\ I_4 \end{bmatrix} \tag{3}$$

and, solving for V_1 and V_4 in terms of I_1 and I_4, gives

$$V_1 = \left(Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right) I_1 + \left(Z_{14} - \frac{Z_{12}Z_{24}}{Z_{22}}\right) I_4$$

$$V_4 = \left(Z_{41} - \frac{Z_{21}Z_{42}}{Z_{22}}\right) I_1 + \left(Z_{44} - \frac{Z_{24}Z_{42}}{Z_{22}}\right) I_4. \tag{4}$$

Substituting (2) and arranging them in matrix form gives

$$\begin{bmatrix} V_1 \\ V_4 \end{bmatrix} = \begin{bmatrix} \dfrac{2Z_{0e}Z_{0o}}{(Z_{0e}+Z_{0o})S} & \dfrac{2Z_{0e}Z_{0o}\sqrt{1-S^2}}{(Z_{0e}+Z_{0o})S} \\ \dfrac{2Z_{0e}Z_{0o}\sqrt{1-S^2}}{(Z_{0e}+Z_{0o})S} & \dfrac{S^2(Z_{0e}-Z_{0o})^2+4Z_{0e}Z_{0o}}{2S(Z_{0e}+Z_{0o})} \end{bmatrix} \begin{bmatrix} I_4 \\ I_4 \end{bmatrix}. \tag{5}$$

Before proceeding further, it is necessary to introduce one additional circuit element, the "unit element" (u.e.), defined by the $ABCD$ matrix as

$$\frac{1}{\sqrt{1-S^2}} \begin{bmatrix} 1 & Z_0 S \\ \dfrac{S}{Z_0} & 1 \end{bmatrix}. \tag{6}$$

The matrix of the unit element is the same as that of a transmission line of electrical length $\theta=\pi f/(2f_0)$ and characteristic impedance Z_0. A series of unit elements has the following interesting properties when inserted between a lumped network and a termination of impedance Z_0.

1) The driving point impedance remains exactly the same as without the u.e.'s,
2) the magnitude of the transfer impedance remains unchanged,
3) the phase of the transfer impedance may be changed.

If unit elements are inserted before the lumped network instead of between the network and its termination,

1) the input and transfer impedances are changed,
2) the magnitude of the reflection coefficient remains unchanged,
3) the phase of the reflection coefficient is changed.

These properties are readily apparent if one thinks of the u.e.'s as lengths of line inserted between a network and its load or between the network and its driving source.

Now consider the cascade of a unit element and an S-plane inductor[10] [Fig. 3(a)]. The $ABCD$ matrix is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{\sqrt{1-S^2}} \begin{bmatrix} 1 & Z_0 S \\ \dfrac{S}{Z_0} & 1 \end{bmatrix} \begin{bmatrix} 1 & LS \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{1-S^2}} \begin{bmatrix} 1 & (L+Z_0)S \\ \dfrac{S}{Z_0} & \dfrac{S^2 L}{Z_0}+1 \end{bmatrix}. \tag{7}$$

The corresponding impedance matrix is

$$[Z] = \begin{bmatrix} \dfrac{Z_0}{S} & \dfrac{Z_0\sqrt{1-S^2}}{S} \\ \dfrac{Z_0\sqrt{1-S^2}}{S} & \dfrac{S^2 L + Z_0}{S} \end{bmatrix}. \tag{8}$$

Comparing this with (5) shows the two to be equivalent, if

$$\text{(a)} \quad Z_0 = \frac{2Z_{0e}Z_{0o}}{Z_{0e}+Z_{0o}} \qquad \text{(b)} \quad L = \frac{(Z_{0e}-Z_{0o})^2}{2(Z_{0e}+Z_{0o})}. \tag{9}$$

Solving for Z_{0e} and Z_{0o} gives

$$Z_{0e} = Z_0 + L + \sqrt{L(Z_0+L)},$$

$$Z_{0o} = \frac{Z_0 Z_{0e}}{2Z_{0e} - Z_0} = \frac{Z_0 + L + \sqrt{L(Z_0+L)}}{1 + \dfrac{2L}{Z_0} + \dfrac{2}{Z_0}\sqrt{L(Z_0+L)}}. \tag{10}$$

Thus the equivalence of the networks shown in Fig. 3(b) has been established where Z_{0e}, Z_{0o}, Z_0 and L are related by (10).

A similar procedure can be applied for other port conditions and equivalent circuits may be developed. A list of all possible circuit configurations and their equivalent S-plane circuits is given in Table I for the case of unequal strip widths. The equal width case is obtained by setting $Z_{0e}^a = Z_{0e}^b$ and $Z_{0o}^a = Z_{0o}^b$.

The synthesis procedure to be used is based upon a series of equivalent circuits known as Kuroda's identities.[11] Kuroda showed the equivalence of the circuits listed in Table II (p. 98). As a sample proof, the equivalence of the networks in Fig. 4 will be demonstrated by

[10] An S-plane inductor L represents the characteristic impedance of a transmission line and has units of ohms.[7]

[11] K. Kuroda, "Derivation Methods of Distributed Constant Filters from Lumped Constant Filters," test for lectures at Joint Meeting of Konsoi Branch of Institute of Elec. Commun., of Elec., and of Illumin. Engrs. of Japan, p. 32; October, 1952. (In Japanese.)

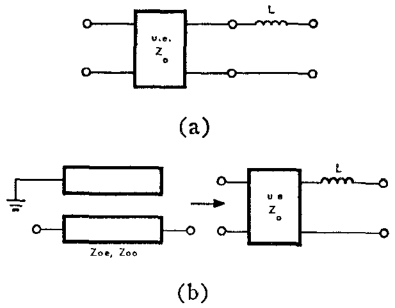


Fig. 3—(a) Cascade connection of a unit element and an S-plane inductor. (b) Equivalence of stripline and S-plane networks.

TABLE I
EQUIVALENT CIRCUITS FOR UNEQUAL STRIP WIDTHS

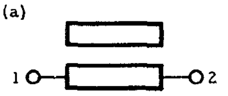
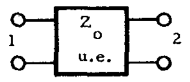
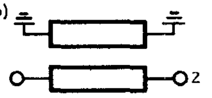
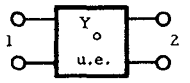
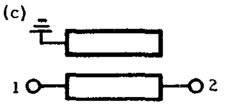
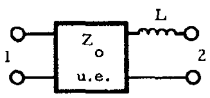
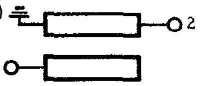
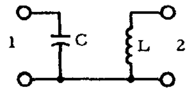
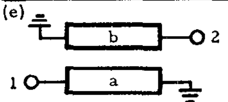
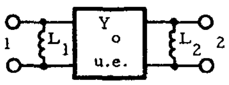
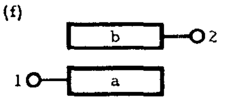
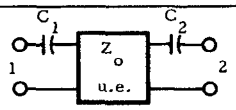
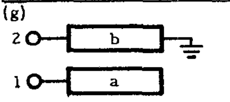
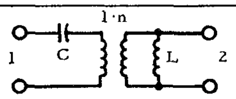
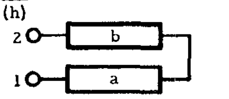
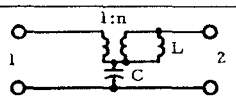
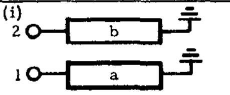
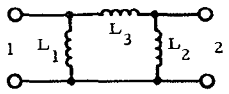
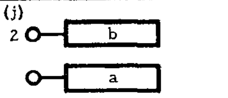
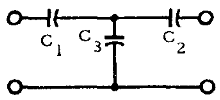
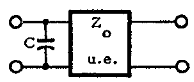
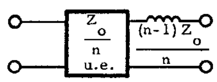
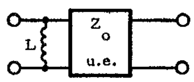
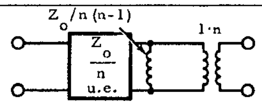
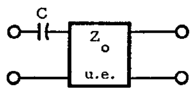
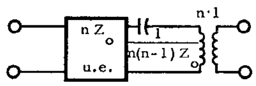
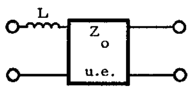
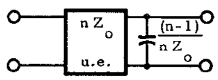
TEM Circuit	Equivalent Circuit	Element Values		
(a) 		$Z_o = \frac{Z_{oe} + Z_{oo}}{2}$		
(b) 		$Y_o = \frac{1}{2}\left(\frac{1}{Z_{oe}} + \frac{1}{Z_{oo}}\right)$		
(c) 		$Z_{oe} = Z_o + L + \sqrt{L(Z_o + L)}$ $Z_{oo} = \frac{Z_o + L + \sqrt{L(Z_o + L)}}{1 + \frac{2L}{Z_o} + \frac{2}{Z_o}\sqrt{L(Z_o + L)}}$		
(d) 		$Z_{oe} = \frac{L}{C Z_{oo}}$ $Z_{oo} = L - \sqrt{L\left(L - \frac{1}{C}\right)}$ $LC > 1$		
(e) 		$\frac{Y_{oe}^{a} + Y_{oo}^{a}}{2} = Y_o + \frac{1}{L_1}$	$Y_{oe}^{a} + Y_{oe}^{b} = \frac{1}{L_1} + \frac{1}{L_2}$	$\frac{Y_{oe}^{b} + Y_{oo}^{b}}{2} = Y_o + \frac{1}{L_2}$
(f) 		$\frac{Z_{oe}^{a} + Z_{oo}^{a}}{2} = Z_o + \frac{1}{C_1}$	$Z_{oo}^{a} + Z_{oo}^{b} = \frac{1}{C_1} + \frac{1}{C_2}$	$\frac{Z_{oe}^{b} + Z_{oo}^{b}}{2} = Z_o + \frac{1}{C_2}$
(g) 	 $n > 1$	$\frac{Z_{oe}^{a} + Z_{oo}^{a}}{2} = \frac{1}{C} + \frac{L}{n^2}$	$Z_{oo}^{a} + Z_{oo}^{b} =$ $\frac{1}{C} + L\left(1 - \frac{1}{n}\right)^2$	$\frac{Z_{oe}^{b} + Z_{oo}^{b}}{2} = L$ $n^2 > LC\ (n - 1)$
(h) 	 $n < 0$	$\frac{Z_{oe}^{a} + Z_{oo}^{a}}{2} = \frac{1}{C} + \frac{L}{n^2}$	$Z_{oo}^{a} + Z_{oo}^{b} =$ $L\left(1 - \frac{1}{n}\right)^2$	$Z_{oe}^{b} + Z_{oo}^{b} = \frac{1}{C} + L$
(i) 		$\frac{Y_{oe}^{a} + Y_{oo}^{a}}{2} = \frac{1}{L_1} + \frac{1}{L_3}$	$Y_{oe}^{a} + Y_{oe}^{b} =$ $\frac{1}{L_1} + \frac{1}{L_2}$	$\frac{Y_{oe}^{b} + Y_{oo}^{b}}{2} = \frac{1}{L_2} + \frac{1}{L_3}$
(j) 		$\frac{Z_{oe}^{a} + Z_{oo}^{a}}{2} = \frac{1}{C_1} + \frac{1}{C_3}$	$Z_{oo}^{a} + Z_{oo}^{b} =$ $\frac{1}{C_1} + \frac{1}{C_2}$	$\frac{Z_{oe}^{b} + Z_{oo}^{b}}{2} = \frac{1}{C_2} + \frac{1}{C_3}$

TABLE II
KURODA'S IDENTITIES

Original Circuit	Equivalent Circuit	n
		$1 + Z_0 C$
		$1 + \frac{Z_0}{L}$
		$1 + \frac{1}{Z_0 C}$
		$1 + \frac{L}{Z_0}$

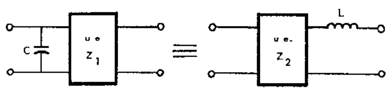


Fig. 4—Kuroda identity.

writing the $ABCD$ matrix. For the network on the left of Fig. 4,

$$\frac{1}{\sqrt{1-S^2}}\begin{bmatrix}1 & 0\\ CS & 1\end{bmatrix}\begin{bmatrix}1 & Z_1S\\ \frac{S}{Z_1} & 1\end{bmatrix} = \frac{1}{\sqrt{1-S^2}}\begin{bmatrix}1 & Z_1S\\ \left(C+\frac{1}{Z_1}\right)S & Z_1CS^2+1\end{bmatrix} \quad (11)$$

and, for the network on the right of Fig. 4,

$$\frac{1}{\sqrt{1-S^2}}\begin{bmatrix}1 & Z_2S\\ \frac{S}{Z_2} & 1\end{bmatrix}\begin{bmatrix}1 & SL\\ 0 & 1\end{bmatrix} = \frac{1}{\sqrt{1-S^2}}\begin{bmatrix}1 & (Z_2+L)S\\ \frac{S}{Z^2} & \frac{S^2L}{Z_2}+1\end{bmatrix}. \quad (12)$$

The two networks are equivalent if

$$\text{(a) } Z_1 = Z_2 + L, \quad \text{(b) } C + \frac{1}{Z_1} = \frac{1}{Z_2}, \quad \text{(c) } \frac{L}{Z_2} = Z_1C. \quad (13)$$

If a parameter n is defined by $n = 1 + Z_1C$, then

$$Z_2 = \frac{Z_1}{n}; \qquad L = \frac{n-1}{n}Z_1. \quad (14)$$

All techniques necessary for design have now been presented. The procedure to be followed is:

1) Choose the function of S to be synthesized that provides the desired response.
2)[12] Perform the synthesis by modern network techniques to yield a lumped network.
3) Insert a series of unit elements of characteristic impedance Z_0 between the lumped network and its load (of impedance Z_0), if the input impedance is to be preserved. If the quantity of interest is a reflection coefficient, unit elements of characteristic impedance Z_0 can also be inserted in front of the network.
4) Use Kuroda's identities to obtain a configuration of unit elements and lumped elements that can be separated into a cascade of circuits similar to those in Table I.
5) From the L, C and Z_0 values determine Z_{0e} and Z_{0o} for each section.

Example—It is desired to design a resistively terminated, Butterworth filter having a reflection coefficient characteristic of order three. The normalized element values for this network, obtained by consulting tables,[2,13] are shown in Fig. 5(a). Since the quantity of interest is a reflection coefficient, unit elements can be added both before the load and after the source. Adding three unit elements of characteristic impedance $Z_0 = 1$, two after the source and one before the load, gives the network of Fig. 5(b). The response of this network differs from that of Fig. 5(a) only in the phase of the reflection coefficient.

Kuroda's identities (Table II) are now applied to the shunt capacitors to obtain a cascade of realizable TEM elements. The transformed circuit is shown in Fig. 5(c).

Consulting the equivalent circuits in Table I shows the above circuit can be realized as shown in Fig. 5(d).

Z_{0e} and Z_{0o} for each section are obtained from the corresponding L and Z_0 in Fig. 5(c). Once Z_{0e} and Z_{0o} are known, the dimensions of the elements can be found from available tables[14–20] or by the method given by

[12] Lumped circuit element values have been computed and tabulated for Butterworth and Chebyshev filter characteristics.[2,13] The tabulated element values must be chosen for the proper characteristic. For example, element values for a Butterworth transfer impedance which assumes an ideal current source are not the same as those for a network having a Butterworth transmission coeficient which assumes a matched source. In general, steps 1) and 2) can be omitted by consulting the many excellent tables available.

[13] L. Weinberg, "Network design by use of modern synthesis techniques and tables," *Proc. Natl. Electronics Conf.*, vol. 12. pp. 704–817; 1956.

[14] S. B. Cohn, "Shielded coupled-strip transmission line," IRE TRANS ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 29–38; October, 1955.

[15] S. B. Cohn, "Parallel-coupled transmission-line resonator filters," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 223–231; April, 1958.

[16] W. J. Getsinger, "Coupled rectangular bars between parallel plates," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, pp. 65–72; January, 1962.

[17] W. J Getsinger, "A coupled strip-line configuration using printed-circuit construction that allows very close coupling," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. 9, pp. 535–544; November, 1961.

[18] S. B. Cohn, "Characteristic impedances of broadside-coupled strip transmission lines," IRE TRANS ON MICROWAVE THEORY AND TECHNIQUES, vol. 8, pp. 633–637; November, 1960.

[19] S. B. Cohn," Thickness correction for capacitive obstacles and strip conductors," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. 8, pp. 638–644; Novermber, 1960.

[20] J. D. Horgan, "Coupled strip transmission lines with rectangular inner conductors," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. 5, pp. 92–99; April, 1957.

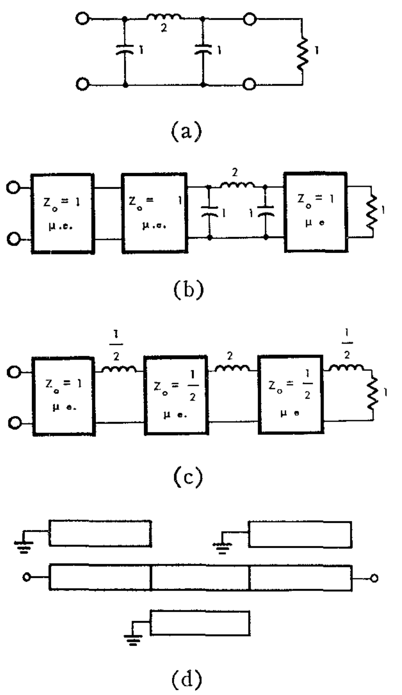


Fig. 5—(a) Third-order Butterworth filter. (b) Butterworth filter with unit elements added. (c) Equivalent circuit after application of Kuroda's identities. (d) Microwave network that realizes a third-order Butterworth response.

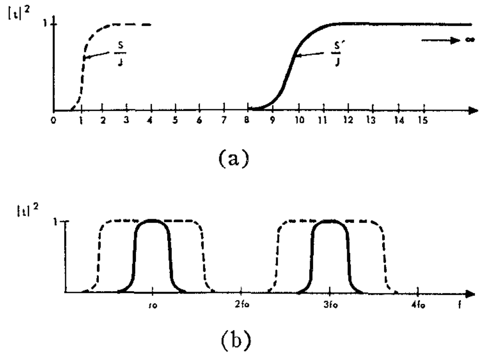


Fig. 6—(a) Renormalized S'-plane high-pass filter. (b) Corresponding f-plane filter.

Ozaki and Ishii for the case of unequal stripwidths.

To make filter computations as simple as possible, as in the above example, most networks are designed with a load impedance level of 1 Ω and a normalized band-edge frequency of $\omega = 1$.

Changing the impedance level of a normalized S-plane network to Z_0 is accomplished by multiplying all normalized resistance and inductance values by Z_0 and dividing all capacitance values by Z_0.

Frequency normalization of S-plane networks cannot be accomplished in the same manner as that associated with lumped elements. The impedance of an S-plane inductor is $Z = jL \tan \pi f/2f_0$. Frequency normalization in the S plane, therefore, can be accomplished only by a change in f_0 which is tantamount to a change in the line lengths used in the TEM structure. For lumped elements, a frequency change is accomplished by multiplying the frequency variable ω by a constant. It is obvious that the analogous S-plane variable S is not multiplied by a constant in the above normalization procedure.

The effect of a renormalization in the S-plane variable $j \tan \pi f/2f_0$ is to change the location of the band-edge response of the network in the real frequency interval between 0 and f_0. Renormalizations of this type are the bandwidth controlling factors in TEM distributed line filters. As an example, consider a normalized S-plane high-pass filter which transforms to a band-pass filter in the f plane (see Fig. 1). This filter has a bandwidth of 100 per cent about the point f_0 and has a response which is periodic in $2f_0$. Assume that a change is made from S to the new variable $S' = 10S$. The pertinent mapping points are now

$$\frac{S}{j} = \frac{S'}{10j} = 0 \quad \text{maps into } f = 0,$$

$$\frac{S}{j} = \frac{S'}{10j} = 1 \quad \text{maps into } f = 0.938 f_0,$$

$$\frac{S}{j} = \frac{S'}{10j} = \infty \quad \text{maps into } f = f_0.$$

The renormalized mapping properties in the S' plane and in the f plane are shown in Fig. 6, together with the responses of the original filter. Thus it is seen that the bandwidth of the distributed TEM filter is controlled by a renormalization of the S-plane variable and the effect of this renormalization is to change the characteristic impedance of the component elements in the TEM structure.

III. Equivalent TEM Filter Structures

When a filter has been designed by the methods of the preceding section and the pertinent element values have been obtained, the problem of constructing the network remains. The design techniques presented do not directly take into account the practical limitations on element values and circuit arrangement. However, judicious use of the techniques can often greatly simplify the construction of the device.

A. Element Values

When working with S-plane capacitors and inductors, it is very important to keep in mind the range of element values that can be realized. With lumped circuit capacitances, element value ranges of 10^6:1 can be achieved easily. In microwave circuits, the available range is much more limited. For example, consider the real frequency TEM circuit realization of an S-plane capacitor which is represented by a length of open circuited transmission line [see Fig. 7(a)].

The constant $1/C$ is associated with the characteristic impedance of the line Z_0. By consulting graphs of Z_0 as a function of line width,[21,22] it becomes apparent that

[21] "The Microwave Engineers Handbook," Horizon House-Microwave, Inc., Brookline, Mass.; 1962.

[22] R. H. T. Bates, "The characteristic impedance of the shielded slab line," IRE Trans. on Microwave Theory and Techniques, vol. 4, pp. 28–33; January, 1956.

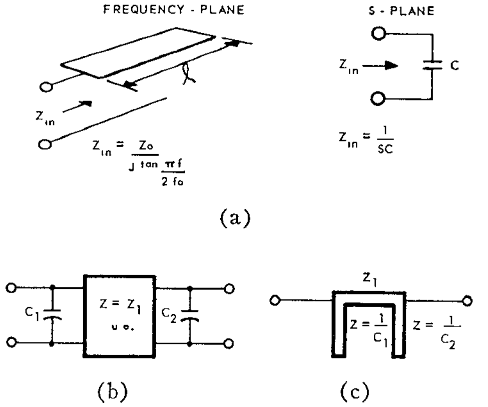


Fig. 7—(a) Stripline realization of S-plane capacitor. (b) S-plane network. (c) Stripline realization.

for $Z_0 < 10$ the lines become very wide and for $Z_0 > 300$ they become extremely narrow. Thus, the realizable range of Z_0, and consequently C, is only on the order of 30:1. As might be expected, this places a severe restriction on the responses that can be obtained with a given circuit configuration. Because of this restriction, the practical realization of a filter with a specified response may appear to be difficult if not impossible. Fortunately, in many cases, it is possible to realize the same response with more than one physical configuration. Before proceeding with the realization problem, it is useful to investigate the behavior of some TEM line sections and to obtain, where possible, different physical forms that realize similar responses.

B. Equivalent TEM Elements

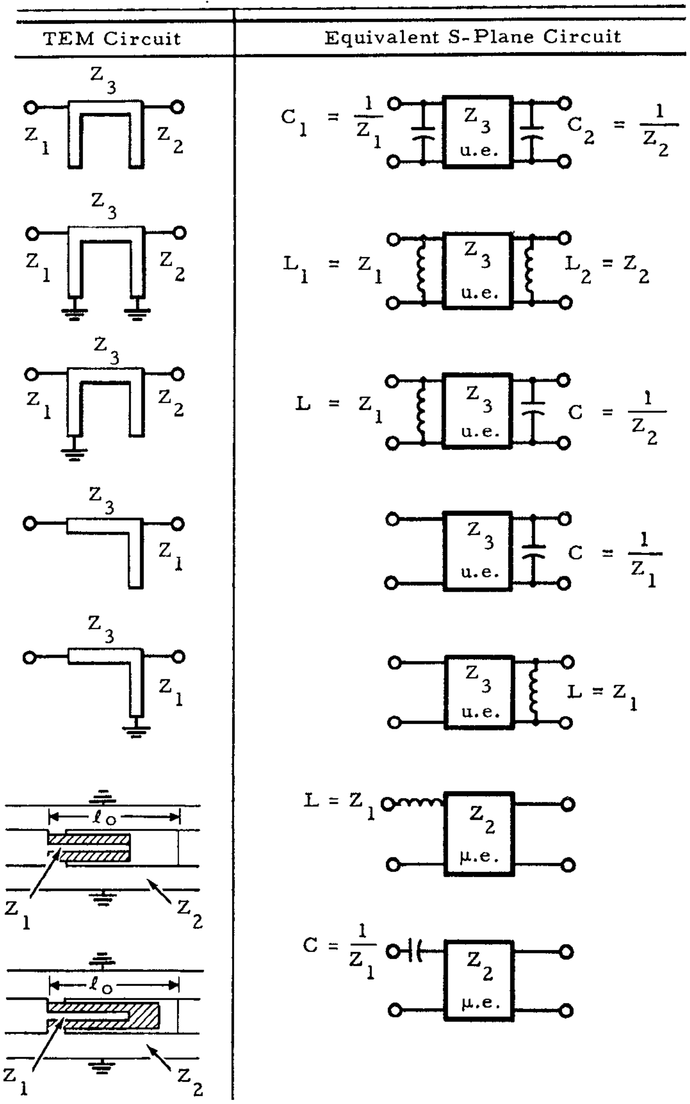
Assume that it is desired to obtain a TEM line realization of the S-plane network shown in Fig. 7(b). There is no simple circuit, as found by consulting Table I, that realizes this network. However, the circuit can be realized in a TEM network as a combination of two shunt capacitors and a series unit element as shown in Fig. 7(c).

Table III lists a number of S-plane equivalent circuits for elements with series and shunt open and short circuited lines. The series stubs are constructed by using either a double coaxial structure or a single coaxial structure between flat ground planes. The impedance Z_2 of the unit element is the impedance of the inner coaxial structure, with respect to the outer conductor, and the impedance of the inner coaxial structure Z_1 is made numerically equal to L for the case of an inductor or $1/C$ for a capacitor. The inner coax is filled with a dielectric such that the series stub will fit within the unit element length l_0. Series stubs are useful in realizing large series capacitors or small series inductors. Construction of the shunt stubs is self-explanatory.

The importance of the equivalent circuits of Table III can be seen if one considers the realization of a unit element followed by an S-plane inductor (see Table I). Application of Kuroda's first identity (Table II) shows this to be the same as a shunt capacitor followed by a

TABLE III

EQUIVALENT CIRCUITS FOR ELEMENTS WITH SERIES AND SHUNT OPEN AND SHORT CIRCUITED LINES



unit element. This configuration can be realized as shown in Fig. 8.

For a filter element of this type to have a broad stop band, a large value of L or C is required. Assume values of $Z_1 = 1$ and $L = 2$. Then it follows that $Z_2 = 3$ and $C = \frac{2}{3}$. Normalizing to an impedance level of 50 Ω gives, for the series line with shunt C, $Z_2 = 150$ Ω and $Z_3 = 75$ Ω. Impedances of this magnitude are within the realizable range and present no problem.

For the parallel coupled line realization, the even and odd mode impedances are

$$Z_{0e} = Z_0 + L + \sqrt{L(Z_0 + L)} = 5.45$$

$$Z_{0o} = \frac{Z_{0e}}{1 + \frac{2L}{Z_0} + \frac{2}{Z_0} L(Z_0 + L)} = 0.55.$$

Normalizing to 50 Ω gives $Z_{0e} = 272$ Ω and $Z_{0o} = 27.5$ Ω. Attempts at realizing impedances of this order in

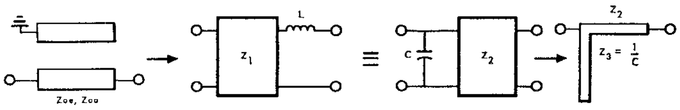


Fig. 8—Stripline equivalents.

parallel coupled lines lead to impractical configurations requiring very small strips having extremely close coupling. Conversely, if it is desired to achieve a narrow stop-band, the parallel coupled element may well prove easier to construct. This is a direct consequence of the limited range of TEM impedance values obtainable.

C. Classification of Elements from Bandwidth Considerations

In designing a filter, bandwidth is a major consideration. For the case of microwave TEM filters, because of the limited range of practical impedance values, it is very important that the designer choose the physical form applicable for a given bandwidth.

The starting point for the bandwidth consideration is the synthesis of the S-plane filter. Filters that are synthesized on a normalized basis will have their cutoff at $S/j=1$ and the corresponding TEM line realization will have a band-edge cutoff at $f_0/2$. In general, the normalized filters will have bandwidths[23] of 100 per cent centered about f_0 and will be periodic in $2f_0$. For changes in bandwidth, the variable S is changed to $S'=S/K$, $(K>0)$ by substituting S' for S in the desired filter function. This changes the band-edge cutoff f_c from $f_0/2$ to

$$\tan \frac{\pi f_c}{2f_0} = K.$$

Then

$$f_c = \frac{2f_0}{\pi} \tan^{-1} K. \tag{19}$$

The fractional bandwidth then becomes

$$\frac{2(f_0 - f_c)}{f_0} = 2\left(1 - \frac{2}{\pi} \tan^{-1} K\right). \tag{20}$$

For $K>1$, the bandwidth decreases and for $K<1$ it increases.

Substituting the variable S' for the variable S changes the original filter elements L and C to the new elements $L'=L/K$ and $C'=C/K$. This means that for narrow bandwidth $(K>1)$, small L's and small C's are required, whereas large L's and large C's are required for wide bandwidths $(K<1)$.

These considerations give a criterion for establishing the bandwidth properties of TEM line configurations. For the case of parallel coupled lines, practical realization requires that Z_{0e} and Z_{0o} be reasonably close to Z_0 (a practical range being $Z_{0e}<150\ \Omega$ and $Z_{0o}>20\ \Omega$). The relations in Table I provide the information necessary to evaluate the bandwidth capabilities of these sections. For example, consider the strip-line element in Table I(c). This section has a stop-band centered about f_0, the bandwidth of which is controlled by the value of L. Inspection of the equations for Z_{0e} and Z_{0o} show that these values are close to Z_0 only when L is small. Thus this section is practically realizable only for $L<1$ and is a good element for use in realizing narrow, stop-band filters. On the other hand, its shunt stub-line equivalent (Table III) can be realized for a wide stop band without encountering difficulty in achieving the element values.

For the case of the element in Table I(f), the equations for Z_{0e} and Z_{0o} show that this section will be difficult to realize for small C's the bandwidth capabilities of all the sections shown in Tables I and III can be determined in a similar manner.

IV. Comparison of TEM and Lumped Element Filters[24]

When a filter is synthesized in the S plane, its response in the real frequency plane is not identical to its S-plane response. An S-plane Butterworth filter, for example, does not have a Butterworth response with respect to the frequency variable ω in its TEM realization.

A. High-Pass and Low-Pass Filters

Theorem: The frequency plane response of a normalized S-plane Butterworth filter whose frequency variable is $\tan \pi\omega/2\omega_0$ provides a better approximation to the ideal low-pass or high-pass filter characteristic than does a frequency plane Butterworth filter whose variable is ω for $0<\omega<\omega_0$.

Proof I: Low-pass filter with band-edge cutoff at $\omega=\omega_0/2$. The Butterworth expression that approximates the ideal characteristic is

$$|t|^2 = \frac{1}{1 + (\text{frequency variable})^{2n}}. \tag{21}$$

The ideal characteristic has a value of 1 in the range $0<\omega<\omega_0/2$. Further, in this frequency range,

$$\tan \frac{\pi\omega}{2\omega_0} < \omega$$

and therefore

$$\frac{1}{1 + \left(\tan \dfrac{\pi\omega}{2\omega_0}\right)^{2n}} > \frac{1}{1 + \omega^{2n}}. \tag{22}$$

Thus the variable $\tan \pi\omega/2\omega_0$ provides a better approximation to the ideal characteristic than does the variable ω. In the frequency range $\omega_0/2<\omega<\omega_0$, the

[23] The term "bandwidth" will be taken as synonymous with "bandwidth about f_0" and can be either a stop band or a pass band.

[24] This comparison is made only over the interval $0<\omega\leq\omega_0$ for the case of low-pass and high-pass filters and over the range $0\leq\omega\leq 2\omega_0$ for band-pass filters. Thus, the repetititve nature of the distributed filter is not considered in the comparison.

ideal characteristic has a value of zero. In this range, $\tan \pi\omega/2\omega_0 > \omega$ and thus

$$\frac{1}{1+\left(\tan \frac{\pi\omega}{2\omega_0}\right)^{2n}} < \frac{1}{1+\omega^{2n}}. \tag{23}$$

Therefore, once again, the variable $\tan \pi\omega/2\omega_0$ provides a better approximation to the characteristic than does the variable ω.

Proof II: The proof for the case of a high-pass filter with band-edge cutoff at $\omega_c = \omega_0/2$ follows in an analogous manner.

B. Band-Pass Filters

The low-pass normalized lumped element Butterworth filter of order n has the transmission characteristic

$$|t(j\Omega)|^2 = \frac{1}{1+\Omega^{2n}}.$$

An unnormalized low-pass filter of bandwidth ω_c then has the characteristic

$$\left| t\left(\frac{j\Omega}{\omega_c}\right)\right|^2 = \frac{1}{1+\left(\frac{\Omega}{\omega_c}\right)^{2n}}.$$

The low-pass to band-pass transformation[4] is

$$\Omega = \frac{\omega^2 - \omega_L{}^2}{\omega}$$

where

$\omega_2 - \omega_1 = \omega_c$ the filter bandwidth,

$\omega_1\omega_2 = \omega_L{}^2$ the geometric center of the lumped element pass-band,

$\omega_2 =$ upper band-edge frequency,

$\omega_1 =$ lower band-edge frequency.

Then

$$|t(j\omega)|^2 = |t|_L^2 = \frac{1}{1+\left[\frac{\omega^2-\omega_L{}^2}{\omega\omega_c}\right]^{2n}}. \tag{24}$$

The TEM line realization of a high-pass S-plane Butterworth filter can be used as a band-pass filter symmetric about $\omega = \omega_0$. For the normalized S-plane high-pass filter,

$$\left| t\left(j \tan \frac{\pi\omega}{2\omega_0}\right)\right|^2 = \frac{1}{1+\left[\frac{1}{\tan \frac{\pi\omega}{2\omega_0}}\right]^{2n}}. \tag{25}$$

For a bandwidth of ω_c about $\omega = \omega_0$,

$$\text{fractional bandwidth} = \frac{\omega_c}{\omega_0} = 2\left(1 - \frac{2}{\pi}\tan^{-1} K\right)$$

[see (20)], then

$$\left| t\left(\frac{j \tan \frac{\pi\omega}{2\omega_0}}{K}\right)\right|^2 = |t|_{\text{TEM}}^2 = \frac{1}{1+\left[\frac{K}{\tan \frac{\pi\omega}{2\omega_0}}\right]^{2n}}.$$

Solving for K gives

$$K = \tan \frac{\pi}{2}\left(1 - \frac{\omega_c}{2\omega_0}\right)$$

or

$$|t|_{\text{TEM}}^2 = \frac{1}{1+\left[\frac{\tan \frac{\pi}{2}\left(1-\frac{\omega_c}{2\omega_0}\right)}{\tan \frac{\pi\omega}{2\omega_0}}\right]^{2n}}. \tag{26}$$

The response of the TEM realization of the S-plane filter is symmetric about $\omega = \omega_0$. The band-edges of both the TEM and the lumped element band-pass filters can be made identical provided that

$$\omega_2 = \omega_0 + \frac{\omega_c}{2}$$

$$\omega_1 = \omega_0 - \frac{\omega_c}{2}$$

$$\omega_1\omega_2 = \omega_0{}^2 - \frac{\omega_c{}^2}{4} = \omega_L{}^2.$$

Defining $B = \omega_c/\omega_0$, the filter fractional bandwidth restricted to the range $0 < B < 1$, and $X = \omega/\omega_0$, in the range $0 < \omega < 2\omega_0$, allows the lumped element characteristic to be written as

$$|t|_L^2 = \frac{1}{1+\left[\frac{1 - X^2 - \frac{B^2}{4}}{BX}\right]^{2n}} \tag{27}$$

and the TEM characteristic as

$$|t|_{\text{TEM}}^2 = \frac{1}{1+\left[\cot \frac{\pi B}{4} \cot \frac{\pi X}{2}\right]^{2n}}. \tag{28}$$

A comparison of the two filters is made by evaluating the characteristic slopes at the low band-edge frequency for fractional bandwidths in the range $0 < B < 1$.

The slope of the TEM filter response at band-edge $X=1-B/2$ is given by

$$\left.\frac{d\,|t|^2_{\text{TEM}}}{dX}\right|_{X=1-B/2} = \frac{\pi n}{2\sin\pi\left(1-\frac{B}{2}\right)}. \qquad (29)$$

The slope of the lumped element filter is

$$\left.\frac{d\,|t|^2_{L}}{dX}\right|_{X=1-B/2} = \frac{2n\left(1-\frac{B}{2}\right)^{2n-1}\left[1+\frac{2}{B}\left(1-\frac{B}{2}\right)^2\right]}{\left[1+\left(1-\frac{B}{2}\right)^{2n}\right]^2}. \qquad (30)$$

Defining $A=1-B/2$, the ratio of slopes is given by

$$\frac{\text{Slope}_{\text{TEM}}}{\text{Slope}_L} = \frac{\pi(1-A)[1+A^{2n}]^2}{4A^{2n-1}[A^2-A+1]\sin\pi A}. \qquad (31)$$

The slope ratio is plotted in Fig. 9. From the graph, it is apparent that the slope of the TEM filter characteristic at band-edge is always greater than the corresponding slope of the lumped element filter, the factor becoming greater as the bandwidth is increased. Both have the same limiting slope for narrow bandwidths ($B\to 0$). The TEM filter has a sharper cutoff slope than the lumped element filter, especially for moderate to wide bandwidths and for a large number of sections.

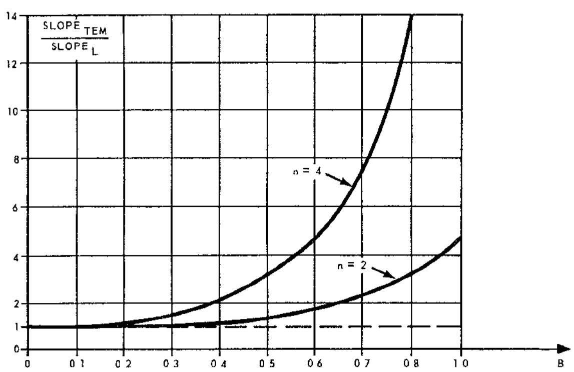


Fig. 9—Ratio of low band-edge slopes for the TEM and lumped elements band-pass filters vs fractional bandwidth for two- and four-section Butterworth network.

V. Applications

A. Equivalence of Stub and Parallel Coupled Line Filters

A common TEM line configuration used in many filters is shown in Fig. 10(a) along with its equivalent circuit. Cohn[15] obtained a design procedure for this type of filter by use of image parameter theory. The methods of the preceding sections can be used to obtain an exact synthesis procedure. Application of Kuroda's identities provide some insight into the synthesis problem. The basic identity to be used is shown in Fig. 10(b). Application of this identity to the end filter section gives the circuit of Fig. 10(c). The transformer is brought out to accessible terminals by transforming each component from left to right sides of the transformer, Fig. 10(d). The network between the transformer and the last unit element has the same general form as the original circuit. Continuation of the above process leads to the circuit of Fig. 10(e). The cascade of transformers can be replaced by a single transformer and the capacitor can be transformed to the input terminals as shown in Fig. 10(f).

The dual network along with its equivalent circuit is shown in Fig. 11(a). Application of a procedure analogous to the one in Fig. 10 gives the dual equivalent circuit shown in Fig. 11(b). The following discussion pertinent to an exact synthesis procedure applies equally well to both the circuit of Fig. 10(a) and its dual Fig. 11(a).

One apparent result of the preceding transformation is that the equivalent circuit, Fig. 10(f), has only $N+2$ degrees of freedom: the N unit elements, the transformer with turns ratio n; and the capacitor C. For the case of symmetric networks, the transformation can be performed in a manner such that $n=1$. This allows the practical realization of these networks in the form of stepped impedance lines with one series or shunt element.

On the other hand, the circuit of Fig. 10(a) contains $2N+1$ parameters but has the same order characteristic polynominal as Fig. 10(f) and is thus a redundant structure. In fact, for many practical filter structures, the N unit element values can be set equal to the characteristic impedance of the system and the $N+1$ capacitors still provide enough degrees of freedom to allow realization of the desired network function. Also, the parallel-coupled filter has no S plane, L-C ladder equivalent, valid over the entire frequency spectrum. Thus, any attempt to obtain an exact synthesis procedure for this type of network by using an L-C ladder prototype will fail.

An exact synthesis of both the circuits of Fig. 10(a) and (f) is possible, but the procedure is not the same as for networks that have an L-C ladder, S-plane prototype. An example of this method is given in Appendix I.

Two important points should be emphasized as a result of the above discussion:

1) From the viewpoint of exactly synthesizing parallel coupled bar filters [of the type in Fig. 10(a) and Fig. 11(a)], the only S-plane elements required are C's, L's, ideal transformers and unit elements. Furthermore, all filters of this type have been shown to require only one L or one C and

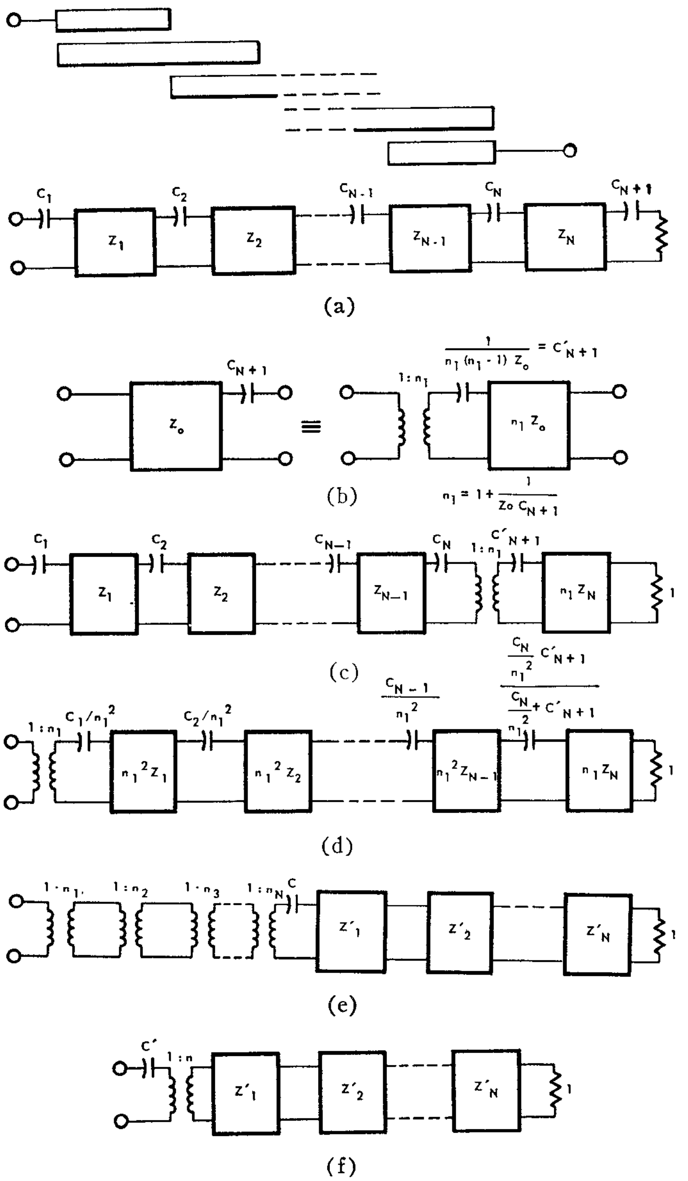


Fig. 10—(a) Stripline circuit and equivalent circuit. (b) Kuroda identity. (c) Application of Kuroda identity to end section of filter. (d) Network obtained by bringing the transformer to accessible terminals. (e) Equivalent circuit after n cycles of transformation. (f) Equivalent circuit with single transformer.

can always be realized theoretically in the alternate forms which do not require parallel coupled bars.

2) By carrying out the synthesis procedure outlined in Appendix I for a large number of cases, tables[25] of element values can be obtained which enable the microwave engineer to design filters of the type shown in Figs. 10 and 11 with known responses over the entire frequency range. There is no bandwidth limitation in the synthesis procedure. The limitations that arise will be due to physical realizability of the required element values.

[25] The many tables now available do not contain the element values obtained by the above synthesis procedure. This is a consequence of the fact that these networks do not have an S-plane, L-C ladder equivalent. The insertion loss function of these networks is of a form different than those with L-C equivalents and thus leads to different element values. See Appendix I (41) and (42) and the following discussion.

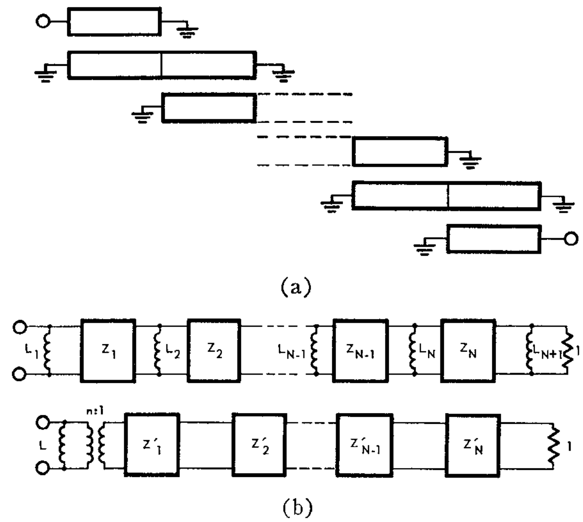


Fig. 11—(a) Stripline circuit and equivalent circuit. (b) Equivalent circuit without parallel coupled bars.

An appreciation of the problem of physical realizability is important. The parallel coupled bar filter, Fig. 10(a); the uniform line with series open stubs; or the stepped impedance filter with one series element, Fig. 10(f), realize the same response. However, for a given bandwidth, one configuration may be more practical from the standpoint of realizable element values.

B. Complementary Filters

In many situations, it is desirable to design a lossless network to couple two resistances over a specified band of frequencies in such a way that the driving source will be properly matched outside the band. Assume that the input impedance of the load and coupling network is Z_1. Then, if another impedance Z_c is added in series such that Z_1+Z_c is constant, the over-all input impedance is resistive and a perfect match can be provided over the entire frequency range. The dual situation is the addition of admittances in parallel such that Y_1+Y_c is constant. Impedances or admittances that add to give a constant independent of frequency are said to be complementary.[26]

Theorem: A lossless ladder network, terminated in a 1 Ω resistor that realizes a Butterworth transfer impedance $Z_{12}(S)$, has a driving point impedance $Z_1(S)$ which always has a complementary impedance $Z_c(S)$ that can be realized in a lossless ladder terminated in a 1 Ω resistor. The network for $Z_c(S)$ can be obtained by replacing the L's and C's of $Z_1(S)$ with $\boldsymbol{C}$'s and $\boldsymbol{L}$'s, respectively, where $\boldsymbol{C}=1/L$, and $\boldsymbol{L}=1/C$.

[26] For a thorough discussion of complementary filters, see Guillemin,[4] p. 476.

As an example, if $Z_1(S)$ is realized in the ladder form of Fig. 12(b), then $Z_c(S)$ is realized as shown in Fig. 12(c). The network definitions are shown in Fig. 12(a).

Proof: For the network of Fig. 12(a), the input power is dissipated in the 1 Ω resistor, so that

$$|I_1|^2 \operatorname{Re}[Z_1(j\omega)] = |V_2|^2.$$

Thus,

$$\operatorname{Re}[Z_1(j\omega)] = |Z_{12}(j\omega)|^2 = \frac{1}{1+\omega^{2n}} \tag{32}$$

for a Butterworth response.[27] From the definition of complementary filters, it follows that

$$\begin{aligned}\operatorname{Re}[Z_c(j\omega)] &= 1 - \operatorname{Re}[Z_1(j\omega)] &(33)\\ &= 1 - |Z_{12}(j\omega)|^2\\ &= 1 - \frac{1}{1+\omega^{2n}}\\ &= \frac{1}{1+\left(\frac{1}{\omega}\right)^{2n}}. &(34)\end{aligned}$$

Substituting $\omega' = 1/\omega$ gives

$$\operatorname{Re}[Z_c(j\omega)] = \frac{1}{1+(\omega')^{2n}}. \tag{35}$$

Thus, the network of $Z_c(S)$ has the same geometry and the change in variable results in converting every L and C of $Z(S)$ into a $\mathbf{C}$ and $\mathbf{L}$ where

$$\mathbf{C} = \frac{1}{L} \qquad \mathbf{L} = \frac{1}{C}.$$

The prototype elements of Butterworth filters have been tabulated in several references[13,28] and, as a result of the above theorem, the complementary network prototypes can be obtained by inspection.

By using the methods described in Section III it is possible to design complementary microwave filters. For practical microwave realization, the series connection required for complementary impedances is in most cases not as desirable as the parallel connection of admittance complements. The complementary admittance lumped element prototypes for a third-order Butterworth response are shown in Fig. 13(a).

For the admittance Y_1, the procedure for obtaining the TEM line realization follows that of the example in Section II. Since the input admittance is to be preserved, unit elements can only be inserted before the

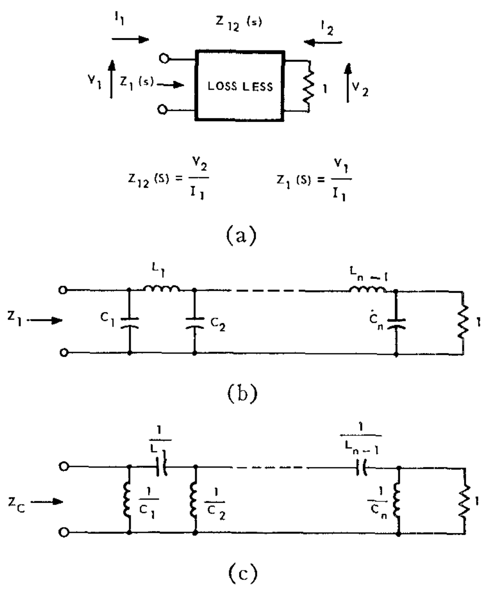


Fig. 12—(a) Network parameters. (b) Butterworth ladder network. (c) Complementary ladder network.

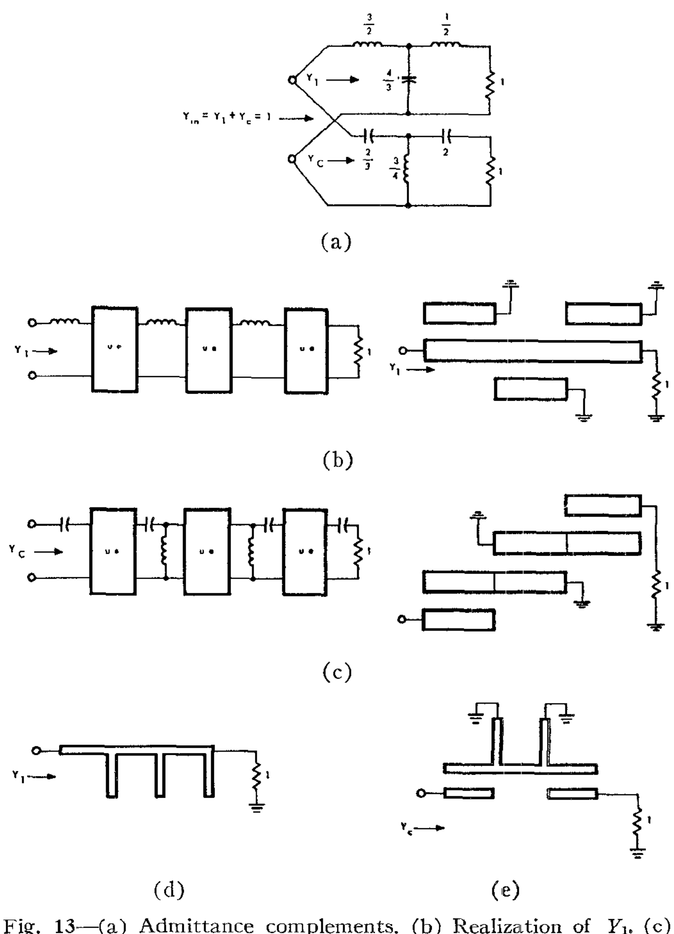


Fig. 13—(a) Admittance complements. (b) Realization of Y_1. (c) Realization of Y_c. (d) Realization of Y_1 without parallel coupled lines. (e) Realization of Y_c with a reduced number of parallel coupled bars.

[27] Note that this filter is designed on a transfer impedance basis and assumes a current source. When the two complementary filters are connected, their common input is purely resistive (R) and provides a matched load to a generator with internal resistance equal to R.

[28] P. R. Geffe, "Computer prepared tables enable design of ultraflat networks," *Electronic Design*, vol. 8, pp. 48–51; August 31, 1960.

load. The final equivalent circuit along with a TEM line realization is shown in Fig. 13(b). A realization of Y_c can be obtained in a similar manner, but will, in general, involve ideal transformers. These transformers can often be eliminated by using a technique illustrated in Appendix II. The final transformerless equivalent circuit for Y_c, along with a TEM realization, is shown in Fig. 13(c).

There are two major drawbacks to the TEM circuits shown in Figs. 13(b) and (c): 1) the parallel coupled sections with shorted ends are difficult to construct and 2) the filter of Fig. 13(b) can provide only relatively narrow stop bands due to the restriction previously discussed. These difficulties can be reduced if the circuits are realized in a different form by using the identities in Table III. Thus, the circuit of Fig. 13(b) can be realized as shown in (d)[29] and that of (c) can be realized as shown in (e).[30] By using the identities in Table III, it is possible to reduce filters with parallel coupled elements to an alternate form.

C. Experimental Results

A number of filters employing most of the network configurations discussed have been built and tested. Excellent agreement with theory was obtained in all cases. For construction details of the stop-band configurations of open shunt stubs [Fig. 13(d)] or parallel coupled bars [Fig. 13(b)], the reader is referred to the paper by Matthaei and Schiffman.[31] Construction details of parallel coupled filters are available in the literature.[32] To the author's knowledge, no construction details of filters employing series stubs have been reported. In view of this, two filters employing these elements, one a band pass about f_0, the other a band stop about f_0, will be described.

The S-plane prototype for the band-pass filter is shown in Fig. 14(a). The element values are those corresponding to a Butterworth reflection coefficient characteristic of order 3 and a bandwidth of 100 per cent. Note that the usual 1, 2, 1 element distribution is not used but rather the reciprocals of these values. This arises from the fact that the 1, 2, 1 values provide a low-pass S-plane response. A band pass about f_0 corresponds to a high-pass S-plane filter. The low-pass to

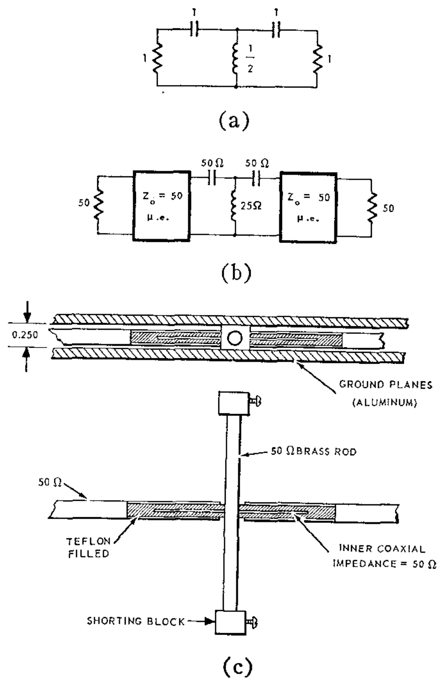


Fig. 14—(a) S-plane band-pass prototype. (b) Prototype after addition of two unit elements and renormalization to $Z_0 = 50\ \Omega$. (c) Construction details of three-section Butterworth filter.

high-pass transformation of a normalized network requires replacing each shunt element by a dual series element with a reciprocal value and each series element by a dual shunt element with reciprocal value. (See Guillemin,[4] p. 602.)

Since the filter was to provide a specified reflection coefficient, a unit element was added before and after the lossless part of the network. Consulting Table III, the network is seen to be realizable as the cascade of a series capacitor stub, a shunt inductor stub and another series capacitor stub. No transformations were required. Normalization to $Z_0 = 50\ \Omega$ gives the final equivalent circuit [Fig. 14(b)].

The filter was constructed between ground planes having a 0.250-inch spacing with a 50 Ω round center conductor. The design center frequency was 2 Gc. The shunt 25 Ω inductor was obtained by using two 50 Ω stubs, one on either side of the center conductor. Construction details are shown in Fig. 14(c). The response and VSWR of the filter is shown in Fig. 15(a) along with the theoretical values. A photograph of the filter is shown in Fig. 15(b). The primary purpose in constructing this filter was to establish the degree to which junction effects would alter the multiple response at 6 Gc. As is evident from Fig. 15(a), these effects were small. Insertion loss in the pass bands was less than 0.4 db and could be further improved by using silver plated conductors rather than brass and aluminum.

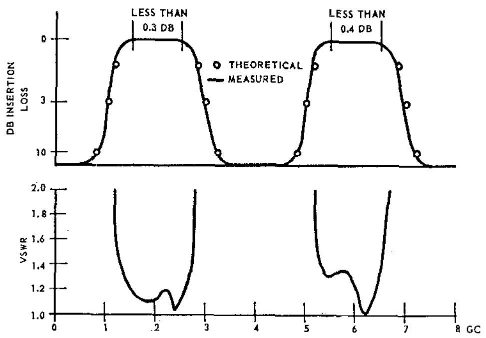
The S-plane prototype for the stop-band filter is shown in Fig. 16(a). The element values are those for a five-section, 0.1-db ripple Chebyshev reflection coeffi-

[29] The equivalence of the networks in Fig. 13(b) and (d) was discussed by Matthaei and Schiffman at the 1963 PTGMTT National Symposium in Santa Monica, Calif. Their paper, to be published in the IEEE TRANSACTIONS under the title "Exact Design of Band-Stop Microwave Filters," provides information relative to constructing these networks.

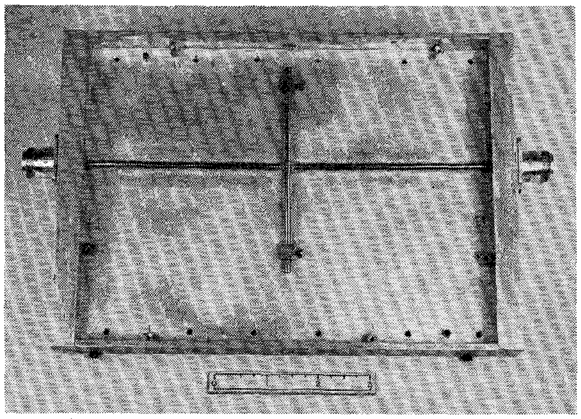
[30] This network can be constructed entirely without parallel coupled bars by using series stubs. See Section V-C.

[31] G. L. Matthaei and B. M. Schiffman, "Exact Design of Band-Stop Microwave Filters," presented at 1963 PTGMTT National Symposium, Santa Monica, Calif., May 20–22.

[32] G. L. Matthaei, "Design of wide-band (and narrow-band) band-pass microwave filters on the insertion loss basis," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. 8, pp. 580–593; November, 1960.

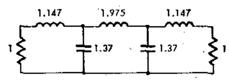


(a)

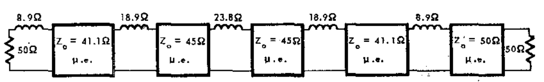


(b)

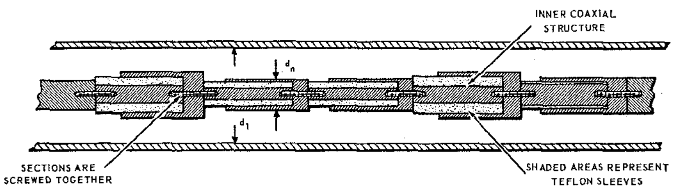
Fig. 15—(a) Insertion loss and VSWR of third-order Butterworth filter. (b) three-section Butterworth filter.



(a)

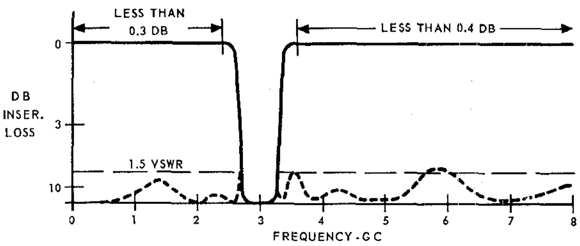


(b)

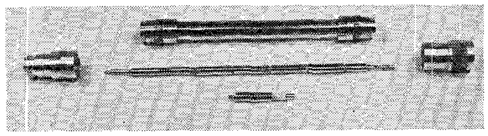


(c)

Fig. 16—(a) 5-section 0.1-db Chebyshev filter prototype. (b) Filter circuit after application of Kuroda's Identities and renormalization to 50 Ω. (c) Construction of stop-band filter with series shorted stubs. The impedance of the unit element is realized by using the correct d_1/d_n ratio and that of the inductors by using the correct inner coaxial impedance.



(a)



(b)

Fig. 17—(a) Insertion loss and VSWR of five-section Chebyshev band-stop filter with series stubs. (b) Band-stop filter.

cient characteristic. The filter was designed to have a 30 per cent band stop about f_0. For a normalized band-edge frequency of $\Omega = 1$, the 0.1-db bandwidth[33] is 100 per cent. Dividing Ω by a constant K, where K is greater than one, reduces the bandwidth in accordance with the discussion in Section III-C. The required constant for a 30 per cent stop band is

$$K = \tan \frac{\pi 0.85 f_0}{2 f_0} = 4.16.$$

Application of Kuroda's identities and renormalization to $Z_0 = 50\ \Omega$ gives the final equivalent circuit of Fig. 16(b).

The filter was constructed using a double coaxial structure. Construction details are shown in Fig. 16(c). The response and VSWR of the filter is shown in Fig. 17(a) and a photograph of the filter is shown in Fig. 17(b).

Little difficulty was encountered in constructing the filters with series stubs. In fact, junction effects for the series stubs were found to be less significant than for shunt stubs, especially when low impedance values were required.

VI. Conclusions

TEM networks designed by modern network theory techniques have been shown to exhibit the following advantages over those based on approximate methods:

1) The equivalent circuits used are theoretically valid over the entire frequency range.
2) Different physical structures that realize identical responses can often be obtained.
3) Limitations are readily found on the response obtainable with a given physical configuration.

[33] A Chebyschev filter characteristics as a 3-db point whose locations depend on the number of elements. For $\Omega = 1$, the characteristic differs from the ideal by a value equal to the specified ripple and thus serves as a convenient bandwidth reference point.

4) The networks, in general, employ a minimum number of elements.
5) It is possible to construct networks that realize exactly a desired input impedance, reflection coefficient or phase response.

The small range of realizable impedance values has been shown to be a limiting factor in the response obtainable in several TEM filter configurations. Equivalent network elements have been demonstrated to allow simplification in physical circuitry as well as improvement in response.

The real frequency response of TEM realizations of S-plane high-pass or low-pass Butterworth filters was shown to be better than that of corresponding lumped element filters. An S-plane high-pass filter, used as a band-pass filter in the frequency plane, was shown to have steeper cutoff at the band-edge than the corresponding lumped element filter. In this comparison, multiple responses of the microwave filter were not considered.

Complementary microwave filters are shown to be useful in providing an exact match of source to load over a wide frequency band. Following the procedure given, the elements of Butterworth complements are easily obtained. Techniques for eliminating ideal transformers in networks with series capacitors and shunt inductors are shown to be useful in obtaining practical circuit configurations. The exact synthesis procedure for parallel coupled line filters can be used to obtain tables that will enable these filters to be designed in a simple manner.

The basic circuit elements required for synthesizing network configurations discussed in this paper are S-plane inductors and capacitors, ideal transformers and unit elements. The parallel coupled bar is not a necessary element; however, it allows a different physical realization for networks involving basic elements.

Application of the basic network theory described in Section III is not restricted to the structures described. Couplers, transformers and many other common microwave components can be analyzed and designed by the use of these methods.

Appendix I

The following example demonstrates an exact synthesis procedure applicable to

1) parallel coupled bars with open ends,
2) parallel coupled bars with shorted ends,
3) stepped quarter-wave sections with one series open stub,
4) stepped quarter-wave sections with one shunt shorted stub,
5) uniform line with quarter-wave spaced series open stubs,
6) uniform line with quarter-wave spaced shunt shorted stubs.

Types 1), 3) and 5) are equivalent and 2), 4) and 6) are their respective duals. The basic prototype to be considered is shown in Fig. 18(a).

The transmission function for a network of this type is[34]

$$|t|^2 = \frac{-S^2(1-S^2)^N}{P_{N+1}(-S^2)} = \frac{\Omega^2(1+\Omega^2)^N}{P_{N+1}(\Omega^2)} \tag{36}$$

where

$$S = j\Omega = j\tan\frac{\pi f}{2f_0}$$

$$N = \text{number of unit elements}$$

P_{N+1} is a polynominal of degree $N+1$ in Ω^2.

In accordance with the discussion in Section V-A, the N unit element impedance values can be arbitrarily assigned. A convenient choice is to set $Z_1 = Z_2 = \cdots Z_N = 1$. The first step in the synthesis procedure is to obtain a suitable form for $|t|^2$.

Letting $\Omega = \tan$, θ gives

$$|t|^2 = \frac{\tan^2\theta(1+\tan^2\theta)_N}{P_{N+1}(\tan^2\theta)} = \frac{\sin^2\theta}{Q_{N+1}(\cos^2\theta)} \tag{37}$$

where

Q_{N+1} is a polynomial of degree $N+1$ having different coefficients than those of P_{N+1}.

Now, letting $x = \cos\theta$, (37) becomes

$$|t|^2 = \frac{1-x^2}{Q_{N+1}(x^2)} = \frac{x^2-1}{(x^2-1) - G_{N+1}(x^2)}$$

where

$$Q_{N+1}(x^2) = G_{N+1}(x^2) + (1-x^2)$$

or

$$|t|^2 = \frac{1}{1 - F_{N+1}(x^2)} \tag{38}$$

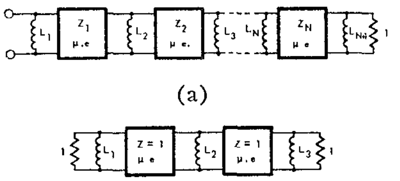
where

$$F_{N+1}(x^2) = \frac{G_{N+1}(x^2)}{x^2-1}.$$

The next step in the synthesis procedure is the approximation problem; that is, a criterion must be established that defines the manner in which the function $F_{N+1}(x^2)$ is to be chosen to approximate the desired $|t|^2$ characteristic. This is, in general, a difficult problem and the reader is referred to literature[2,4,5] for detailed discussions.

A common approximation is the maximally flat response which is obtained by setting all but the $(N+1)$st

[34] This can be obtained by mutiplying the $ABCD$ matrices for the network.



(a)

(b)

Fig. 18—(a) Prototype for 1) parallel coupled bars with shorted ends, 2) Line with quarter-wave short circuited stubs. (b) Prototype for third-order Butterworth filter.

coefficient of $F_{N+1}(x^2)$ equal to zero. Applying this criterion gives

$$|t|^2 = \frac{1}{1 - \frac{Gx^{2(N+1)}}{x^2-1}}$$

where

G is the $(N+1)$st coefficient of the polynomial $G_{N+1}(x^2)$.

The prototype for a third-order maximally flat filter is shown in Fig. 18(b).

There are two unit elements; therefore, $N=2$. Then

$$|t|^2 = \frac{1}{1 - \frac{Gx^6}{x^2-1}}.$$

Since

$$x^2 = \frac{1}{1-S^2},$$

$$|t|^2 = \frac{1}{1 - \frac{G}{S^2(1-S^2)^2}}$$

or

$$|\rho|^2 = 1 - |t|^2 = \frac{G}{G - S^2(1-S^2)^2}. \qquad (39)$$

The bandwidth of the filter is determined by the constant G; using $S=j \tan \pi f_c/2f_0$ and setting $|\rho|^2=0.5$ allows (39) to be solved for G. The fractional bandwidth of the filter is

$$\frac{2(f_0-f_c)}{f_0}.$$

The general network configuration of the filter is known and it is sufficient for synthesis purposes to obtain the input impedance that corresponds to the given $|\rho|^2$. For the details of this procedure, see Guillemin,[4] p. 458.

For $G=36$, the bandwidth is 70 per cent and

$$Z_{in}(S) = \frac{S^3+4S^2+7S}{S^3+4S^2+7S+12}. \qquad (40)$$

The synthesis procedure is different depending upon which of the possible configurations is desired. Richard's theorem,[7] basically common to each procedure, is used to remove unit elements.[35] This theorem states that if $m_1m_2 - n_1n_2|_{S^1}=0$, then a unit element of impedance Z_{in} (1) can be removed with the remaining impedance being

$$Z'_{in}(S) = Z_{in}(1)\frac{SZ_{in}(1) - Z_{in}(S)}{SZ_{in}(S) - Z_{in}(1)}.$$

Under these conditions, both the numerator and denominator of $Z'_{in}(S)$ contain the factor (S^2-1) and $Z'_{in}(S)$ is thus one degree lower than $Z_{in}(S)$. The product of the even parts of numerator and denominator, respectively, of $Z_{in}(S)$ is given by m_1m_2 and the product of the odd parts is given by n_1n_2. A one stub network can be obtained by applying Richard's theorem to (40).

$$m_1m_2 - n_1n_2|_{S=1} = 4S^2(4S^2+12) - (S^3+7S)^2|_{S=1} = 0.$$

A unit element of impedance $Z_{in}(1)=\frac{1}{2}$ can be removed. The input impedance of the remaining network is

$$Z'_{in}(S) = Z_{in}(1)\frac{SZ_{in}(1) - Z_{in}(S)}{SZ_{in}(S) - Z_{in}(1)} = \frac{S^2+2S}{4S^2+14S+24}.$$

The common S^2-1 factor has been cancelled. $Y'_{in}(S)=1/Z'_{in}(S)$ has a pole at zero that can be removed by taking out a shunt inductor,

$$L = \frac{1}{12}; \qquad 2S+S^2 \overline{\left)24+14S+4S^2\right.} \quad \frac{12}{S}$$

$$\underline{24+12S}$$

$$2S+4S^2.$$

The remaining impedance is $Z''_{in}(S)=(S+2)/(4S+2)$ after cancellation of the common factor. Applying Richard's theorem to $Z''_{in}(S)$ shows that the remaining impedance consists of a unit element of impedance $\frac{1}{2}$ and a 1-Ω resistance. The final circuit is shown in Fig. 19(a).

To obtain the equivalent three-stub network with unit elements of impedance $Z_0=1$, partially remove a pole (shunt inductor) of $Y_{in}(S)=1/Z_{in}(S)$ so that the remaining input impedance $Z'_{in}(S)$ satisfies $Z'_{in}(1)=1$. This insures that the unit element with $Z_0=1$ can be removed and thus reduces the order of $Z_{in}(S)$ in accordance with Richard's theorem.

For example, using (40),

$$7S+4S^2+S^3 \overline{\left)12+7S+4S^2+S^3\right.} \quad \frac{a}{S}$$

$$\underline{7a+4aS+aS^2}$$

$$(12-7a)+(7-4a)S+(4-a)S^2+S^3.$$

[35] This theorem is discussed further by Grayzel, "A synthesis procedure for transmission line networks," IRE Trans. on Circuit Theory, Vol. CT-5, pp. 172–181; September, 1958.

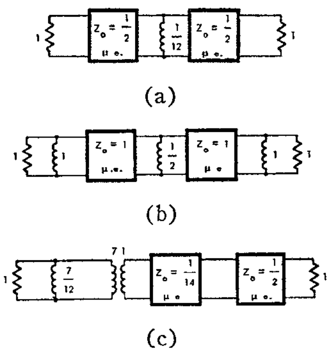


Fig. 19—(a) One-stub third-order Butterworth filter. (b) Three-stub third-order. Butterworth filter. (c) Third-order Butterworth filter synthesized in the equivalent form of Fig. 11(b).

Then

$$Z'_{in}(S) = \frac{S^3 + 4S^2 + 7S}{S^3 + (4 - a)S^2 + (7 - 4a)S + (12 - 7a)}.$$

For $Z'_{in}(1) = 1$, $a = 1$; therefore, $L = 1/a = 1$.

The unit element of $Z_0 = 1$ can now be removed from $Z'_{in}(S)$. The above process is now repeated using the remaining impedance. The final network is shown in Fig. 19(b).

A third form of the network can be obtained by noting that the network must have an equivalent circuit with a shunt inductor, a transformer and a cascade of two unit elements as demonstrated in Section V-A. For

$$Z_{in}(S) = \frac{S^3 + 4S^2 + 7S}{S^3 + 4S^2 + 7S + 12},$$

removal of the shunt inductor (pole of Y_{in} at zero) gives

$$\begin{array}{r|l} & \dfrac{12}{7S} \\ \hline 7S + 4S^2 + S^3 & 12 + \; 7S + \; 4S^2 + S^3 \\ & 12 + \dfrac{48S}{7} + \dfrac{12S^2}{7} \\ \cline{2-2} & \qquad \dfrac{S}{7} + \dfrac{16S^2}{7} + S^3 \end{array}$$

or

$$L = \frac{7}{12}.$$

Then

$$Z'_{in}(S) = \frac{S^2 + 4S + 7}{S^2 + \dfrac{16S}{7} + \dfrac{1}{7}} = \frac{7S^2 + 28S + 49}{7S^2 + 16S + 1}.$$

The turns ratio of the transformer can be obtained by noting that

$$\lim_{S \to 0} Z'_{in}(S) = n^2$$

[see Fig. 19(c)]. This gives $n^2 = 49$ or $n = 7$. Removal of the transformer yields

$$Z''_{in}(S) = \frac{\dfrac{S^2}{7} + \dfrac{4S}{7} + 1}{7S^2 + 16S + 1}.$$

Application of Richard's theorem shows the remaining network consists of a cascade of two unit elements of impedance $Z = 1/14$ and $Z = \frac{1}{2}$ terminated in a 1-Ω resistor. The final equivalent circuit is shown in Fig. 19(c).

The networks of Fig. 19 are exact equivalents. The network of Fig. 19(b) can be realized as a line with shunt stubs or with parallel coupled bars by using the relations in Table I. Elements for filters with a Chebyshev characteristic can be obtained in a similar manner by applying a suitable approximation criterion to (38).

The filters discussed above are said to have a "maximally flat" or "Chebyshev" characteristic, but these responses are not the same as those of networks that have an S-plane, L-C ladder equivalent. The basic equation for a maximally flat filter with an L-C prototype is

$$|\rho|^2 = \frac{1}{1 + \Omega^{2n}} \tag{41}$$

whereas, for the filters discussed above, the basic equation is

$$|\rho|^2 = \frac{1}{1 + \Omega^2(1 + \Omega^2)^{n-1}}. \tag{42}$$

It can be shown that filters of the type described by (41) have steeper band-edge slopes than those whose characteristics are given by (42). In the pass band of the filters ($\Omega \gg 1$), (41) becomes nearly equal to (42). Thus an L-C ladder can serve as the prototype for networks with a characteristic given by (42), but the response realized follows that of the prototype only for f in the vicinity of f_0. Whereas this approximation may lead to a suitable filter, it cannot be used to realize a response that is specified over the entire frequency range.

Appendix II

S-plane ladder networks with series capacitors and shunt inductors generally will have ideal transformers in their TEM realizations. These transformers can be consolidated and transposed to the input or output terminals where they can be realized in a section such as that shown in Table I(g). In many cases, however, it is possible to eliminate the ideal transformers. As an example, consider the lower half circuit in Fig. 13(a) that realizes Y_c. Adding three unit elements of characteristic impedance $Z_0 = 1$ to Fig. 13(a) gives the circuit of Fig. 20(a). Application of Kuroda's identities gives Fig. 20(b). Transforming the 4/3 capacitor to the right hand side of the second transformer, consolidating the trans-

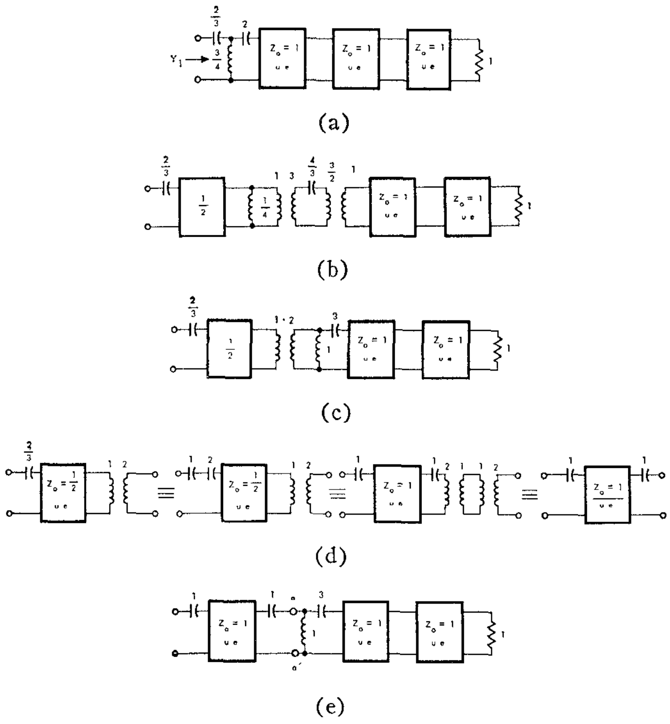


Fig. 20—Elimination of ideal transformers.

formers and transforming the 1/4 inductor to the right hand side of the resultant transformer gives Fig. 20(c). The transformer can be eliminated, if the series 2/3 capacitor is divided into a series combination, so that the capacitor closest to the unit element, when transformed by Kuroda's third identity, yields a transformer with a 2:1 turns ratio or

$$n = 2 = 1 + \frac{1}{Z_0 C} = 1 + \frac{2}{C},$$

$$\therefore \; C = 2.$$

The procedure is illustrated in Fig. 20(d) and the resulting circuit is shown in Fig. 20(e). A similar procedure can be applied to the circuit to the right of terminals aa' to obtain a realization without transformers. For the case of a shunt inductor, the inductance is divided into a parallel combination of inductors so that the required turns ratio results.

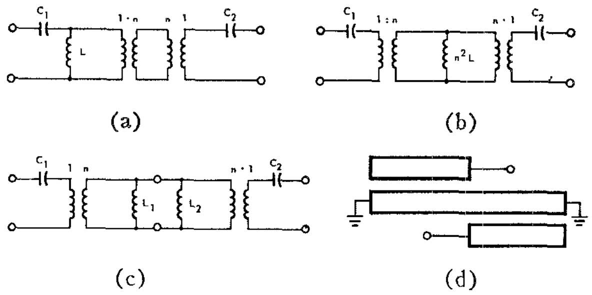


Fig. 21—Realization using sections with ideal transformers.

The above admittance function can also be realized in another way. The circuit of Fig. 20 is left unchanged, if a pair of back to back ideal transformers of turns ratio n is introduced as shown in Fig. 21(a). Transforming the inductor to the region between transformers gives Fig. 21(b). Dividing the inductor, so that

$$\frac{L_1 L_2}{L_1 + L_2} = n^2 L,$$

gives the circuit of Fig. 21(c) which can be realized as the cascade of two elements of the form shown in Table I(g) where n is chosen to make the realization practical. The TEM realization is shown in Fig. 21(d).

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